§1 Preliminary Considerations

In order to represent the knot in more formal and algebraic theories, the mathematicians have tended to disregard how a presentation of the knot is necessary to its existence. The mathematician V. Jones outlines the project of the contemporary formal theory of knots in the following manner:

Although the formalism (for mechanical statistical models or "vertex models") is quite general and not tied to braid presentations or induction, it is still hampered by the need for a two-dimensional projection (shadow) of a three-dimensional object. Our main reason for doing this work was as a step towards a useful and genuine understanding of three-dimensional invariants. So far we have not succeeded. The situation is the same as the poor prisoners in Plato’s allegory of the cave. (Jones, 1989)

The project is coherent, though we believe it confused with regards to its aim of positing a realm of ideal formal entities beyond their appearance on a surface. The desire to bypass the problem of the presentation is an attractive idea that would bring knot theory much more in line with its more formal and 'abstract' cousins such as number theory, algebraic topology, etc. My only reservation is that it cannot be done since it bypasses a special existence of the knot that requires a reference to the two-dimensional projection. More than one commentator from the scientific community has noticed the problem without quite figuring out what to do about it:
As for the Jones Polynomial and its generalizations, these deal with the mysteries of knots in three-dimensional space. The puzzle on the mathematical side was that these objects are invariants of a three-dimensional situation, but one did not have an intrinsically three-dimensional definition. There were many elegant definitions of the knot polynomials, but they all involved looking in some way at a two-dimensional projection or slicing of the knot, giving a two-dimensional algorithm for computation, and proving that the result is independent of the chosen projection. This is analogous to studying a physical theory that is in fact relativistic but in which one does not know of a manifestly relativistic formulation—like quantum electrodynamics in the 1930’s. (Witten, 1989)

To rescue the knot from a formal abyss, we begin by including its presentation as crucial to its identification and existence. We do this not by including the knot within the theory of relativistic physics, but within a theory of the signifier.

A historical and methodological reference may be helpful here: the linguist Roman Jakobson recalled how Einstein’s theory of physical relativity was directly influenced by Jost Winteler’s theory of linguistic relativity:

Winteler remained true to the principle of ‘configurational relativity’ (Relativität der Verhältnisse) that had been disclosed in his dissertation with special reference to the sound pattern of language. In particular his theory required a consistent distinction between the relational invariants and variables within language, respectively termed ‘essential’ and ‘accidental’ properties. According to Winteler’s insight, speech sounds cannot be evaluated in isolation but only in their relation to all other sound units of the given language and to the linguistic functions assigned to them in such a manifold. (Jakobson, 1972)

To construct the diagram interior to a theory of knots is to show how the signifier conditions a variable identity and existence. In order not to confuse the contingent aspects of the signifier with the ‘relativity’ of physics, we could refer to the use of the knot in the history of techniques: the knot is not so much an object but a tool or ornament that is reliant upon its context and inscription. Or one could simply refer to an indiscernible of the linguistic signifier that is only identifiable via the ‘artifice’ of writing: Once upon a time, a bald heir who wore a wig to hide the fact that he had no hair tripped down the stairs while holding his pet hare. The problem of linguistic relativity becomes which hit the floor first? The /her/, /her/, or /er/? Strictly speaking, the spoken signifier /x/ is only an invariance across a variation of written letters.

Such examples, far from being trivial, go straight to the point when attempting to identify what is One and distinguish it from what is the Same and Different through the use of a trait and writing. It is evident that unless one makes a reference to the signifier the problem of identity cannot be formulated; while without writing the problem cannot be resolved.
I want to show that a knot theory that takes its reading and writing seriously—not as something simply to be pushed through or a mere means of de-coding and coding, but as fundamental to problems of identification and existence—is nothing other than an introduction to the problem of structure. Contrary to Jones, and the majority of workers in the field for whom the diagram is a mere sign or index of a formal object in space, we will show how the diagram is a condition for the existence and identification of the knot in its structure.

We abbreviate the problem of how to consider the knot as the structure of a diagram—and not merely as object in space—with the phrase Generalized Placement. We situate our approach not simply within the current theory and practice of Lacanian analysis, but profiting from such works as A. Tarski and A. Robinson. If the latter can write, “If we are willing to accept the idea that a mathematical structure consists of a set of statements, we may even identify a structure with its diagrams” (Robinson, 1956), it is because the problematic difference between diagram and object is far from being an extra-mathematical concern of physics or linguistics; on the contrary, it can be shown to go to the heart of mathematics as such once the difference between numeral and number is put into play. ¹

1.1§ Clinic and Experiment

Historical Background

If a theory is as sound as the problems it resolves, then the correlation of the Generalized Placement Problem with the clinic is straightforward: Freud distinguished between a premier narcissism that is the identification with an image and a secondary narcissism that separates from the image in an object choice. It is this passage from premier to secondary narcissism, from diagram to object, from mask to actor that is at stake. Hence, Freud’s claim that, “The ego is first and foremost a bodily ego, it is not merely surface entity, but a projection onto a surface” (Freud, 1923) prefaces the Mirror Phase of the early Lacan.

Yet it would be grossly unfair to Lacan to say that this is where he leaves the problem, even though the virtues of mirrors have been extolled for the last 40 years by child psychiatrists, at least in France, who have included little mirrors in their waiting rooms alongside coffee tables with magazines. Though neo-Lacanians have been removing every trace of topology from the practice of analysis, others have been working to get it out of the waiting rooms.

Lacan first formulated the generalization of the mirror phase into an experimental context in terms of a Topology in Extension. Mythologically, this generalization may be called a ’Screen Phase’ in the sense that it is the projection of a real image on a surface or concave mirror that is not simply that of the virtual image in a flat mirror. The most evident presentation of the problem was first given by Lacan in optical
models that were only developed later in a purely topological presentation of surfaces. Of course, Lacan still developed the mythology and literary references, even if they remained 'tongue in cheek.' For example unlike a philosophical conception of mimesis—or the actor and her role—Lacan insisted that Freudian identification could be described as a mask that you cannot quite take off: it has an investment or charge of Libido that 'glues' it to the subject: a lamella that results in the horrifying creation of the Hommelette.

In either case, optically or mythologically, an explication of narcissism requires a theory of identification presented in terms of a screen and not a child reflecting on its image in front of a mirror. Indeed, the term Mirror Phase, as it is commonly understood by the psychologically minded, presupposes the very identity of the child that the theory is supposed to explain. No doubt, if left at this level, Lacan's Mirror Phase is nothing more than a frustrated account of origins that any myth attempts to explain: if Saint-Denis walked to the Louvre with his head on a platter, what showed him the way? Beyond the comic relief provided by such paradoxes of representation, what is important is not the specific content of Freud's Narcissism and Lacan's Mirror Phase, but how they approached the problem of differentiating object and image.

Lacan's lasting contribution was an insight into what would be required to introduce a rigor into such problems yet did not depend on an unshakable 'Grund' or quest for certainty. On the contrary, it is a search for a clear account of the basic notions of a discipline such that one can construct a theory in practice, experimentally, in the clinic, and not in a speculative philosophy.

Topologically, the generalization of the mirror phase requires distinguishing Mirror Symmetry from Duality (§4): it is well known that symmetries are determined on axes determined by points of duality. But this insight in turn, depends on a clear and novel account of what is involved with the space of a mirror: it is the fundamental space of a Torus.

In Seminar XII, Crucial Problems for Psychoanalysis (1964-1965), Lacan had indicated more than once that the toric generalization of the mirror phase occurs by turning the torus inside out. Thus, providing the mirror symmetry necessary for the passage from premier narcissism to secondary narcissism. Yet, what is rarely explained well is that this turning inside-out of the torus is a 2-dimensional template for the Borromean requiring an articulation of Duality.

Interdependence of Clinic and Experiment

We take the position that Lacan achieved the theory of Freud experimentally in a topology. By experiment, I do not simply mean what comes to formulate the laws of Nature under a positive science, but a collection of written laws and constructions used in the refutation and corroboration of conjectures—by which I mean, a group
of statements coming to bear not simply—or primarily—on the transmission of a knowledge (savoir), but the distillation of an ignorance.

An experiment may proceed by the use of hardware in the impossibility of a direct observation or human ability, but experimentation need not be limited to such instruments: in linguistics one commonly refers to the 'commutation test' as purely experimental procedure for identifying the sounds (phonemes) and significant units of a language.

Moreover, an experiment need not be empirical. A. Koyré has shown how the imaginary plays an important part in the history of scientific thought since experimentation is never a simple application onto the real. Rather, from Galileo’s ‘discourse experiments’ to Mach’s ‘thought-experiments’ (Gedankenexperimente), an experiment bridges a gap between empirical fact and theoretical concept through an effective use of the imaginary.

Lacan’s experimental topology does not present an over-arching philosophy, complete methodology, or analytic worldview, but a scattered series of problems that appear to be constructed in ignorance of a precise solution. Yet, a work by zig-zags or ‘bricolage’ could only be viewed negatively if it were assumed that a psychoanalytic discourse could be homogenized by a school, religion, or philosophical-scientific community trying to say and refer to the same thing. If one proceeds experimentally, then the problem of presentation, ignorance, and discontinuity comes to the fore. So much so, analysts, at least since Lacan, can no longer simply refer to the clinical cases of a notable analyst and attempt to apply the results in the clinic as if they were simply memorizing a recipe of a chef.

Habitually, within the sciences, the medical clinic is viewed as a place to apply a theory and confirm hypotheses that were tested and verified experimentally in a laboratory. By analogy, a psychoanalytic clinic could be viewed as a place to apply a theory and confirm a psychoanalytic theory, if it were not for the fact that the majority of analysts have absolutely no way of doing a psychoanalytic experiment if it is not on people to begin with. Thus, in therapeutic analysis, the interdependence between clinic-experiment-theory has collapsed and the professional analyst is merely left with applying generalities or cooking strategies onto others using the buzzword ‘clinic.’ This is not to deny that certain therapeutic effects may occur in such games or that one could not write a psychoanalytic cookbook claiming to be an ‘Introduction to the Clinical Lacan.’ Rather the problem with such approaches is that the passion transferred onto psychoanalysis is never adequate to distill its ignorance beyond a question of taste.

Enter the experimental topology of Lacan.

The crucial problem of psychoanalysis is not to develop its practice and theory by reading books and applying ideas to people in the clinic, but to develop what one thinks one (mis)understands of the theory in a practice experimentally. It is not in asking ‘What is Narcissism?’ that one finds the response in science, philosophy,
myth, politics, literature, or in a clinical case. Only a psychoanalytic scholar could think so. For the question itself can only be formulated by establishing the experimental conditions for the functioning of something like a mask, a real image, and the deduction of their consequences in the practice of a theory.

That there does not exist today any qualification of competency in the application or construction of a psychoanalytic theory does not mean there is not a just constraint of discourse by which anyone who claims to practice a cure may be at the same time asked to do more than talk. Indeed, without an experimental dimension it is difficult to see how psychoanalysts could imagine assuming responsibility for their discourse on any other level than a juridical sanction coming to bear on a professional obligation—and not a practice of the theory itself. One discovers here how the scruples of a too professional idea of analysis becomes aligned with a philosophy that finishes by annulling Lacan’s topology in the same way they trivialize the clinic in the argument that topology is only theory while the clinic is just therapeutic practice. If one abstains from an experimental entry to psychoanalysis, its experience will never be adequate: its topology confused with theory and its clinic with therapy. To counter such an all too common reaction, the problem of imprudence in psychoanalysis should receive a more precise elaboration. Until then, Lacan has left us with what could only be a call for a reform of understanding: “What the ideology of contemporary psychoanalysis suffers from is the lack of an adequate topology.”

Once Lacan achieves Freud’s theory of narcissism in an experimental topology, the references to biological and cultural analogies may be dropped (or saved for university conferences). Yet, topology does not relate to the main body of psychoanalytic theory, but to that practice of psychoanalytic observation called reading. Such an experimental manner of proceeding is not only slow, but encounters equivocations and impossibilities that do not lend itself to general introductions. That this lack and contingency would not be read, developed experimentally, or receive a just writing (or mathematics), but would be rejected, has a parallel clinical development. W. Bion wrote:

Ordinary language is already more than a method of recording and enables work to be done, like mathematics, in the absence of the object worked on, it is less effective in precision and universality. So long as our communications are reliable only in the presence of the objects we study we work under disabilities analogous to those of the psychotic and quality must suffer correspondingly. (W. Bion, Transformations, 41-42)

It is not a question of asking whether a psychoanalytic community or institute would be a group of psychotics, but on what conditions a passage to the act in the name of a psychoanalytic theory is, if not avoidable, then treatable experimentally at the level of the clinic—and not through a policing of a government or science.

In an essay entitled “Therapy, Experimentation, and Responsibility,” Georges Canguilhem recounts the difficulty that any medical practice faces the moment the
anxiety of experimentation and technique is bypassed in reserving the exercise of a profession to only those having a university diploma in exclusion of the 'empirics': no *Jus impune occidendi*, following the principle *Fiat Experimentum in corpore vili*. To remain at this level, the government can and must require that any practice come under the judgment of a medical police, while the laws of a theory, once constrained to the laws of nature, must be carried out under reason and policed by philosophy and science. In both cases, what is bypassed by such regulations is a real of technique and experimentation that only medicine, and by implication psychoanalysis, can resolve: the limit between the beneficial and the harmful varies case by case since in medicine one must experiment, that is to say, "one only cures in trembling."

To establish the conditions of the cure, in medicine or psychoanalysis, can only mean to truly have an experience of the other, not as alter-ego or forms of empathy-antipathy, but in the concern for the singularity of the other through experiment. To proceed otherwise amounts to making the patient anonymous under the *a priori* of a theory whose practice is reduced to attaining the other’s singularity only in a transference: through a passion counter any clinical and experimental approach. Canguilhem summarizes the problem thus:

To assume the lesson of clinical experimentation is to accept the formidable moral and intellectual exigencies. The unconsciousness of too many doctors today is not a misrecognition, but on the contrary an indirect recognition by one of the mechanisms of flight and forgetting whose elucidation constitutes a trait of Freud’s genius.⁶

Drawing attention to the importance of experimentation in purifying the ignorance and imaginary of the subject, Lacan writes:

For the knowledge accumulated in the analyst’s experience concerns the imaginary which experimentation constantly runs up against to the point of coming to regulate its allure on the systematic exploration of the imaginary in the subject. […]

In this respect, *experiment* privileges neither the so called “biological” tendency in analytic theory, which of course has nothing biological about it except the terminology, nor the sociological tendency sometimes referred to as “culturalist.” […]

In truth, if analysis borders closely enough on the scientific domains thus evoked that certain of its concepts have been adopted by them, these concepts are not grounded in the *experiments* of those domains.⁷

If today it has become possible to understand how the subject is not the ego and differentiate the two by making analogies to the transcendental subject of philosophy or the alienated subject of language and a literary practice, this is not to resolve the analytic problem since the resurgence of the ego is not responded to by ridicule or by assimilating it to the experiential clinic of therapy. On the contrary, what such
strategies of misreading bypass is how an analytic experiment provides neither an experiential nor an empirical condition, but an ethics: a de-ontology of the clinic whereby a transfer—or passion of ignorance—can be constructed, separated from, and not merely lived.

Forty years ago, Lacan signaled that this experience-experiment signals a ‘beyond’ of psychoanalysis that is the place of the future analyst (Lacan, The Four Fundamental Concepts of Psychoanalysis. The position put forward by this article is that future is what concerns us here today.

2§ Generalized Placement

It is often said (Kauffman, 1983, 1987) that a theory of knots is about the placement problem: How is an object X put in a place Y by the function h:

1) h: X————————>Y

If X = a closed curve S1 and Y = three dimensional space or R 3, and the function h is injective (for each point in R 3 there is at most one point in X that corresponds to it), then we can speak of placing S 1 into R 3 and draw its diagram as:

Thus, a formal knot theory may said to begin with the classification of h, that is to say, how h embeds S 1 into R 3. Yet what this way of proceeding forgets are not only the properties that are not interesting for the embedding of a formal knot by h (its color, texture, etc.), but those scriptural traits and letters that allowed the knot to be identified in diagrams in the first place. For example, the first embedding that took place was the way the Cartesian coordinates (x, y, z) were oriented on the page. To reinsert this forgotten place or parameter back into the theory we must generalize the placement problem by recognizing a double: there is not only a question of how h places a space X into a space Y, but how f places a knot—or space—X in a diagram D. Thus:
Generalized Placement Problem

X: $S^1 = \text{closed curve } f: \text{presentation/transcription}$

Y: $R^3 = \text{three-dimensional space } g: \text{representation/interpretation}$

D: $= \text{diagram } h: \text{embedding of object}$

In mathematics, this way of systematically setting up a space is called a category and includes the various ways a mathematical territory may be mapped as functions, or more generally morphisms, between spaces. For instance, in knot theory mathematicians can be said to work in the category of Differential Topology and Piecewise Algebraic Topology. Depending on the researcher’s epistemology, either they are fine with abstract knotted mathematical objects or they suppose that the object is physical and the formal mathematical theory is only an abstract representation. In either case, it is the referent that is knotted, while it is only the diagram that represents this knottedness in a secondary way—as a convention, as commodity, or indeed, a mere map. Thus, the ancient of history of knots as design or ornamentation may be of interest, but like the diagram itself, it is viewed as a mere cultural artefact.

I want to begin otherwise: by bringing out how the putting into place of the knot-diagram is itself productive of knotting; not as culture or an aesthetic aid, but as constitutive of the knot itself.
2.1§ A Peculiar Problem from the Contemporary Mathematical Theory of Knots

Let us landmark the problem that insists between diagram and object with a concrete example from the mathematical theory of knots itself. The two knot diagrams below are each of the same 10 crossing knot but the one on the right is in a non-alternate presentation having 14 crossings, while the one on the left is the alternate presentation having 10 crossings.

Call the unknotting of the knot the reversal of over and under—a homotopy—at a crossing of a diagram such that through one or more reversals the knot is untied, i.e. reduced to a mere closed circle. It is important to note that in order to define the unknotting number of the knot \( u(k) \) we must refer to the diagram \( D \), and not simply the object in space. Thus:

\[ u(k) = \text{the minimal amount of crossings changes on } D, \text{ i.e., } u(D) \]

What is surprising is that the more complicated knot diagram of 14 crossings undoes with less untying, i.e. with fewer homotopies. Both knot diagrams are of the same knot and can be undone by a reversal of over and under at a crossing, but the knot with more crossings undoes with fewer reversals (=2), while the one with fewer crossings undoes with more reversals (=3). This difference can be expressed by saying that the crossing number (= the minimal amount of crossings in a diagram) does not determine the existence of knotting as defined by the unknotting number (= the minimal number of reversals of over and unders at a crossing sufficient to produce the unknot).
In short, what this difference implies is that the existence of knotting is dependent upon a projection and cannot be established as mere object in space.

If we define the unknotting number $u_{\text{min}}(k)$ on the minimal crossing diagram, we can write this in terms of an inequality:

$$u(k) \leq u_{\text{min}}(k)$$

Which states that the unknotting number is less than or equal to the unknotting on the minimal crossing diagram. Where in the above diagram, $u(k) = 2$ and $u_{\text{min}}(k) = 3$.

There are two important problems here:

1) if we can undo a knot with one homotopy $u(k) = 1$, then it can be shown that $u(k) = u_{\text{min}}(k) = 1$ and its converse.

2) However, if $u(k) \geq 2$, then we are unable to show that it is equal to $u_{\text{min}}(k)$. That is why we write, in general: $u(k) \leq u_{\text{min}}(k)$.

What this result states is that unknotting problem for proper knots, i.e., knots with one component, may be complete (decidable and discernible); while knots that can only be undone with two or more homotopies introduce a certain incompleteness (undecidable/indiscernible) problem.

We will not have time in this paper to examine the unknotting of single string proper knots or problems of completeness/incompleteness. Rather we will concentrate on the problem of linking and locking, then return in Part II to re-examine proper knots. For the moment, it suffices to call attention to the curious detour that is introduced when referring to both diagram and object: strictly speaking, the unknotting number of a knot is not an invariant of space since it requires a presentation in two-dimensions. Thus, to truly recognize knotting one must not simply appeal to the sameness and difference of its form, but two seemingly opposed aspects: its existence and diagram.

Our contention is that this bi-modal entry into the theory of knots, both object and image, knot and diagram, three and two dimensions, opens up an experimental dimension of knot theory. That this experimental dimension would be indicative of an incompleteness inherent to the field requires that we not simply prove theorems in a formal knot theory, but construct diagrams in a structural knot theory. The work becomes clinical, in the analytic sense of the term, to the degree that this bi-modal presentation can be shown to be translatable into the same difficulties that the analyst encounters in his or her practice of speech and language.

2.2§. A Formal Versus an Informal Structural Knot Theory

We call a theory of knots ‘abstract’ and ‘formal’ when there is an extreme parametrization of the knot such that there is essentially only one way $h$ to place one
knot (up to a movement) in the space $Y$ since the diagram correspondence $f: X \rightarrow D$ is assumed to be the same as the object once $g$ is assumed fixed, that is to say, once the object is already supposed to be in space and the diagram is merely a question of representing it. One way to show this is to assume that the map at $D$ is unique—a standard presentation—so that now all the different ways of placing $X$ in the space $Y$ are exactly all the ways of representing it in the diagram $D$.

If $h = (g \circ f)$, then the placement $h$ is forced to go to the unique point to which $g$ does. If we relax the parameterization of $X$ by $D$ and $g$ so that that the diagram at $D$ is no longer the same as the thing at $Y$, then we have an introduction to a structural theory and the foundations for an experimental and clinical theory of knots. At which point there is the suspicion that $g$ may not be univocal: one may have a trompe-l’oeil, anamorphosis, or Kandinsky effect where the difference between object and image is isolated. In such a case, $g$ no longer responds to the question of how to represent $Y$ in $D$, but how to interpret $Y$ in $D$. Said otherwise, at this point, as we will show, there is both a problem of indiscernibility and decision crucial to articulate within the theory of knots itself.

3§ Introduction to Linking & Borromean

The standard formal theory of knots and links takes the position that the planar graph is a projection of the knot and then calls a diagram a projection in which the overs and unders have been established at each crossing by Maxwell’s right hand rule:
What is interpreted in the diagram as 'over' is the 'height' of a string that is represented in the diagram by a broken trait. One of the oldest knot invariants in the history of knot theory is the crossing number: for any knot there is a minimal number of crossings (Tait, 1877); but to find this minimal number we first have to find a way of identifying what the knot or link is in theory.

Let us confine ourselves to a classical theory of links, that is, multi-component closed curves in space. The objects of the theory are equivalence classes [L] of presentations where each class is determined by a movement. Confining ourselves to the theory of links, the equivalence classes [L] are link presentations that are invariant across the Reidemeister moves or isotopy:

To say the link is invariant across an isotopy is to say that the link creates an obstacle to the movement of isotopy in space. The diagram itself viewed in this theory as a commodity of presentation—an illustration—of what is happening in three dimensions.

Each object L, or equivalence class of presentations [L], is named by a number called its linking number: L = lk(x,y).

In a proof, this name, or formula, should designate an object uniquely—two different objects cannot have the same name—and exhaustively. The Standard Definition: Let L = a ∪ b be a link of two components, then let Σ(a, b) be the sum of crossings with a and b.

Lk(a,b) = (a U b)1/2 where ‘a’ and ‘b’ denote the sum of the set of signed crossings {+1, -1}

To show the linking number is unique and complete, it suffices to label each crossing of the Reidemeister moves with the corresponding {+1, -1} then show that it remains invariant in the sense that lk = 0 with regard to any sliding in the plane. For example, the second Reidemeister move is:
Once this is achieved, then the crossings are labeled on any multi-component diagrams to verify that the sum of crossings is not equal to zero:

$$\Sigma(-1 - 1 + 1 - 1)/2 = -1$$

It may be verified that there is a finite series of Reidemeister moves that undo tangling just until the obstacle of the link. We write this finite series of moves: $$R_2(R_1(L)) = -1.$$
However, there are a few different problems with this method. One is well known and the other less so.

1) First, there are multi-component closed curves that are connected but have a linking number of zero.

\[ \text{Link: } L(x,y) = 1 \]

The figure on the right is what Tait called a Lock and J. Milnor has called trivial and almost trivial Homotopy Chains. Two of the most famed examples being the Borromean and the Whitehead:

\[ \text{Lk} = 0 \]

In such cases, the movement of isotopy no longer functions to identify the connection holding the components together. What is required is a higher order movement.
called homotopy such that that Locks (at least some of them) can be identified as obstacles to homotopy.

Defining a homotopy intuitively as the change of crossing sign on one component:

We can now define new types of object that poses a resistance to the movement of homotopy. Notice that a link as such, poses no obstacle to a homotopy because homotopies are only defined on one and the same string.

So then, if a configuration of closed curves has a linking number of zero, then there are still ways it can be connected. These fall into a combination of three main groups:

1) the trivial homotopy chains like the Whitehead above that undo with one homotopy;

2) the non-trivial like the Borromean which does not undo with any homotopies, but only in the removal of one of the components;

3) the almost trivial that do not undo with one homotopy, but with several on the condition that you change the presentation.
With (2) and (3) there emerges different types of homotopy chains that generalize the Borromean:

3.a.) Those that do not undo with one homotopy on one component, but several different components (Sourry Generalization):

2.a ) Those that do not undo by any homotopy or the removal of one component of the configuration, but will undo with the removal of more than one component (Penney Generalization).

4) Those that are a mix of the above types (Large Generalization).
Our goal is not to classify the different types of Borromeans, but to use their characteristic properties to account for the problem that is introduced by this higher order manner of connecting and by problems of presentation.

The introduction of homotopy chains effectively introduces, without ever pointing directly at it, a threefold problem:

a) the complementary space of a Borromean is defined by the fact it is not simply "complementary": this can be intuitively stated by stating that the connection is holding the rings together without ‘borrowing’ the holes of the other component, i.e., it is not linking.

b) this non-complementarity of the negative space of the Borromean can be called duality in a sense that will be explained shortly;

c) the unlocking of the Generalized Borromeans is dependent on their projections; that is to say, a change of presentation can determine an augmentation or subtraction of the number of moves needed to disconnect the components.

A direction for future work must explain the correlation between non-complementary duality, projective sensitivity, and un-doing a configuration-knot, link, or lock.

§§ The Mirror Phase: Problems of Symmetry & Duality

Two things are identical if and only if they share all the same properties. Before we look at the claim itself, does anyone note anything funny? How can two things be identical?

Max Black

Oh you look like John.
I am John!
No wonder you look like him.

Old Timer’s Joke

I will pay more attention than a standard presentation to how the descriptive statements of a knot theory may be put into logical form. The reason I do so is that the symmetry relations between antecedent and consequent, respectively, p and q, in the statement (p implies q) will be important to account for once we begin to construct a logic of narcissism.

The standard way of proceeding begins with two: if it can be shown that any two link diagrams $D(k_1)$ and $D(k_2)$ are the Same (equivalent), then it can be said that the two links ($k_1$, $k_2$) are One (identical).
Call this manner of reading an object ‘Leibnizian’ since it follows his celebrated principle of the *identity of indiscernibles.* Or in everyday language, how the Same is One.

If we are in the theory of Links, two may be the same object in the sense that they belong to an equivalence class \([L]\) of diagrams that is nothing other than their *link-type.*

Essentially, any link-knot theory is seeking to prove the following type of conjecture:

1) If \(D(L_1) \iff D(L_2)\), then \((L_1=L_2)\)

Call this the unidirectional formula (UI), where the reasoning goes from *equivalence* of diagrams to an *equality*—or invariance—of *objects.* In the theory of Links, the predicates \(D\) account for the movements—the Reidemeister moves—by which two things can be considered the same or different in the space \(R^3.\)

In general, \(L_1\) and \(L_2\) are said to be of the same *isotopy type* if there exists an isotopic deformation of \(R^3,\) an equivalence relation such that the class of movements are equal, i.e., \([L_1] = [L_2]\). Said more simply, in the classical theory of links, links \(L_1\) and \(L_2\) are said to be equal if they can be defined as an equivalence class of some particular representative Link.

The converse, however, is false: it is possible to have links that are equal, but do not belong to the same *isotopy type,* if we define *isotopy type* as having the same linking number and attainable by the Reidemeister moves. This is the classic case of Kant’s symmetric objects and the Parmedian paradox of the One is the Same.

For example, the two oriented links below are equal since they are once-linked closed curves, but they neither have the same sign, nor can they be deformed into each other by isotopy. Thus, they do not have the same *isotopy-type* according to (1) above.

This has led the standard theory of links to call such opposed pairs ‘mirror images,’ ‘symmetric objects,’ or ‘left and right-hand objects.’

Assuming the classic terminology, the oriented link is the same as its mirror image but not of the same *isotopy type* since one cannot be transformed into the other through Reidemeister moves.

Thus, we are in the difficult but familiar situation, at least since Kant, of two objects that are the same, but not super-imposable.
What is assumed in classical knot theory is that one can use this non-superimposition to define a subgenre of equivalence: there are those objects that can be made equivalent to their mirror and those that cannot. If they can, then the two objects are called *amphichieral*. If they cannot they are called *chiral*.

In each case, to determine the identity of one object, one is left with trying to make two objects the same—The Same is One—instead of asking how the converse, how the One is the Same opens up a more difficult problem of identification and *Duality*.

The classical theory of knots is dependent upon a picture of the world and is as much a strategy of approach as it is mathematics: it is a *Leibnizian* question of beginning each time with Two and letting a rigorous definition of what is the Same about the One escape by the door. I will follow the tradition for a bit longer since I will not have time to return in future chapters.

The classical *Leibnizian* theories always begin with Two: if the Two are equivalent, but not isotopic through a series of Reidemeister moves, then one can still insist that they are equal (one, identical) if there is a continuous transformation \( h \) that reverses a voluminous object of the embedding space such that: \( h: (x,y,z) \rightarrow (x,y,-z) \).

At the end of this continuous transformation or *flype*—think of pulling a glove inside out where you reverse everything but a finger—if the orientation and sign of the link matches its ‘mirror’ then the two are called ‘amphichieral,’ if not ‘chiral.’ The once linked oriented chain above is, therefore, *chiral*, as it is not transformable onto its ‘mirror.’ For instance, some have called the two oriented links above chiral, while the unoriented Listing knot is *amphichieral* because it can be transformed onto its flat mirror image by the transformation just described:

![Diagram of knots](image)

1.  
2.  
3.  
4.  
5.

But there is a problem: let the reader pick up any book on knot theory to discover that the sequence depicted above is called a orientation preserving continuous transformation (homeomorphism) that turns a left handed 4-crossing knot to a right-handed one. As such the Listing is an amphichieral knot. Depending on the text, the knot depicted in figure (5) will be called the ‘mirror image’ of (1), or they will be called ‘symmetric objects’; while some even suggest that it is a question of merely reversing all the overs and unders of (1) to achieve the mirror image in (5).

Yet the careful reader will notice that the black dot placed in the diagram does not correspond to a mirror reflection of (1) into (5). Further still, to achieve a so-called
'mirror-image' transformation, (i) has to be rotated by a quarter-turn to (5). This quarter-turn rotation is not a Reidemeister move and is nowhere detailed as crucial to the establishment of a mirror image. This problem is dire enough to receive a construction in Part II of our introduction.

What can be stated at this point is the following: what has occurred in the transformation from (1) to (5) is that the knot has been ‘flyped’ in the sense that one pulls a glove inside out. Thus, the correlation between L₁ and L₂ is not what is seen in a flat mirror, this only reverses the direction you look from, but a more subtle ‘perversion’ reversing the voids and the surface distinction of the diagram. Such a transformation is, strictly speaking, a problem of duality. The suggestion that either knot corresponds to a ‘mirror-image’ of the other must be put on hold until a more careful investigation of perversion has been brought out (See §6).

Let us state from this point forward that the continuous transformation from 1. to 5. results in a Dual Presentation L**.

Put a surface on any closed curve in the plane, using the rule that every time you cross an arc change a color from a binary set of opposed colors {+, -}. Then put the put it onto a sphere and perform a flype transformation h(x, y, z) → (x, y, -z). We discover two things:

a) the 4-Listing knot has been turned inside out or ‘flyped.’

b) the result at 5. only appears to be virtual mirror images of 1. In fact it is not, as the correspondence of zones is not exact while the whole configuration at 4. requires a rotation through a quarter turn to go to 5.

By calling the diagram what it is, a Dual presentation that is only secondarily one of symmetry, we must refine our method and rethink the correlation to the symmetric image.
The problem of Duality in each case is not a question of the orientation of the component, but the transposition of the void/surface distinction.

Currently, at least to this author and a few others, there is a grand confusion in the mathematical literature in not distinguishing Duality from Symmetry. Just a few examples:

1) The author of a classic introduction to knot theory defines amphichirality without reference to the arrow orientation of the component; elsewhere he defines amphichirality with reference to the arrow orientation of the components.

2) A well-known mathematician defines links as chiral with reference to the arrow orientation of components, then uses non-continuous transformations between linking numbers to explain why Möbius bands have mirror images (strictly speaking, intrinsically the Möbius band is neither right nor left).

3) The Jones polynomial is said to distinguish a left from right hand trefoil, but only works on oriented diagrams.

Though probably none of these difficulties would be seen as a ‘problem’ by the classical theory of knots, for us they point towards a difficulty in the construction of its foundations. The work of such authors is not what is in question; there have been remarkable results from the formal theories and methods they have invented. However, each time that the identification of the knot is raised, questions of symmetry emerge whose successful treatment depends more on an overriding theory, largely an algebraic method, than the knot.

No doubt, there are other examples where my confusion insists, but in each case it is difficult to ascertain at what point in the mathematical literature:

1) duality problems are confused with symmetry;
2) chirality problems are confused with orientation by arrows;
3) placement in diagrams is assimilated to placement in space;
4) structure is confused with form;
5) thing is assimilated to object.
What follows are a series of exploratory constructions to bring out these problems. If we begin by orienting a link with arrows, then two of the $22/4$ projections are dismissed as being the 'same,' while the two others must be called chiral—since no space perversion will allow the orientation of one link to turn into that of the other, i.e. the linking number does not change its sign or undo no matter how you twist it around through a continuous transformation. This also implies that the link is an object independent of its diagram. However, if one does not orient the links by arrows, then we have another theory in which links can be chiral or amphichiral by isotopy.

![Chiral and Amphichiral Diagrams]

More to the point, if we do not orient the link by arrows, it becomes dependent on its projection, that is to say, through a change of presentation via Reidemeister moves the once linked chain $-1$ transforms to $+1$; whereas in the case of the twice non-oriented linked chain $-2$ no amount of movement will change it to $+2$.

Call the first case, amphichiral and the second chiral with regard to isotopy. We will return to this problem shortly.

Neither vocabulary nor method have been standardized with regard to these problems in the mathematical theory, or if it has, at least to this author’s taste, it is done prematurely in trivializing what is really at stake. I will simply state the problems before returning to put forward a new framework for its consideration.

a) First, in the classical theory, it is an abuse of language to speak of the difference between image and object: in each case, the transformation is between object-object or image-image, but not object-image.

b) Second, in the classical theory, it is an abuse of language to speak of a mirror: rather there is an ‘ideal mirror’ which is silvered on both sides.

c) Third, chirality, or ‘handedness’ is only defined by default, by what is not amphichirality. The assumption that a condition of amphichirality and
continuous transformations is primary causes one to lose sight of the internal difference and discontinuity of the One. \textsuperscript{19}

In the desire to main a conception of invariance based on (1) above, the \textit{identity of indiscernibles}, classical knot theory begins with Two and the Same then runs into predictable difficulties when it encounters a One that is the Same.

This problem jumps to the forefront when the theory is faced with the problem of symmetric objects that no longer respond to any mode of continuous transformation except by default and by an abuse of language.

Of course, I could not imagine that these problems will be worked out for me or others overnight, but what we can do is present a theory of knots and links that accounts for these gaps by engaging a wider audience of listeners to weigh in on the problems.

We call our approach anti-Leibnizian for several reasons, but one of the most apparent is that that we do not begin with Two then seek to determine how they are the Same, but with One and seek to determine who it is the Same. That this 'Sameness' would be cloven, or dual, is an indication of the constructive problem.

I will explain this starting point shortly.

Another reason for insisting on an \textit{anti-Leibnizian} approach is the insistence on examining the converse of proposition (1) above:

\textbf{2) If} (L\textsubscript{1}=L\textsubscript{2}) \textbf{⇒ D(L\textsubscript{1}) ⇔ D(L\textsubscript{2})}

Call this converse of the principle in (1) above: the \textit{indiscernibility of identity} or what some call a \textquote{principle of substitution salva veritae} \textsuperscript{20}

The converse of (1) above leads to a series of well-known problems. \textsuperscript{21}

\textit{Problem #1:}

Two things may be the same with regard to a predicate (or equivalence relation), but still not equal or substitutable. Thus, the converse of (1) does not always hold. This is Kant’s celebrated critique: a left and right hand can be equivalent in all their properties by (1), but not congruent in space, i.e., (k\textsubscript{1}=k\textsubscript{2}) \Rightarrow D(k\textsubscript{1})\Rightarrow D(k\textsubscript{2}). Again, the contemporary mathematical theory of links speak of two links as having exactly the same properties with regard to their linking since they both have a linking number of 1, but they are differently oriented.

\textit{Problem #2:}

Two links may be equal, but we may not yet have discovered how many Reidemeister moves it will take to make them so. That is to say, if we cannot determine an upper-bound on how many Reidemeister moves it will take to transform one
link into another, then there is no finite characterization of linking; i.e. the linking number is not a complete invariant.

Thus, if we remain with the attempt to define a link simply by the linking number and isotopy alone, then not only do we run into (1) the problem with mirror image links that have different isotopy types, but we run into (2) incompleteness: there may be no limit to determining when Two that are the Same will become One.

If left in such a situation, a theory of links remains vague. What is required is a precise way to both identify and determine the existence of linking. It is our position that only by considering the often forgotten converse proposition (2) that the problems can be clarified. I will show an introductory approach starting in section §6 then isolate a conjecture and proposal that has already been made in the field of analysis.

If we can show that the unidirectional formula (UI) is not simply unidirectional, i.e., it has a valid converse, then we can determine a complete link invariant just as we can speak of the converse of the identity of indiscernibles being completed with the indiscernibility of identity. If possible, this would constitute a complete and formal theory of links and knots.

In general, a knot-invariant is stated in the UI formula as:

If two links $k_1$ and $k_2$ are equivalent by diagrams, then their invariants are equal.

Or more precisely:

If two link-types $[k_1]$ & $[k_2]$ are equal, then their invariants are equal.

That is to say,

If $c_1[k_1] = c_2[k_2] \Rightarrow c_1 = c_2$

It is considered complete, if the converse is valid, or if we can construct a model in a higher-level theory in which it would confirmed.

It is crucial to our work to show how in knot-link theory the general invariance formulas above are only unidirectional: that is to say, an incompleteness and an undecidable emerges when the converse and contrapositive are introduced. If in knot-link theory one can only speak of uni-directional proofs, then there are incompleteness problems present: unlike classical geometry, there are no complete invariants in knot and link theory. The reason for this is the insistence of duality problems that not only introduce the necessity of including both diagram and object into the theory, but of rethinking how the problem of identity is established in the first place.

Such an incomplete knot theory must make room for a certain ‘informality’ in the sense that whereas one had been searching—in vain—for complete invariants, there is a more realistic approach: write the indecisions and problematic identities into the theory not as a ‘error’ or ‘vagueness,’ but as the precision required for a con-
struction of the difference between image and object in a mathematical construction. We call this new theory *A Structural Knot Theory*, then show how it gives rise to a Generalized Placement Problem forming the basis of the clinic.

5§ A Love Triangle of the Butcher’s Wife

The Link is not simply an object \( L \) embedded into a space \( h: L \rightarrow R^3 \) but has a place in its Diagram of diagrams \( f: L \rightarrow S^L \). To situate the link in its diagrams is to determine its structure:

![Diagram](image)

If \( f \) is a morphism determining the inscription of \( L \) in the place of Diagrams \( S^L \) and \( h \) a morphism determining the embedding of \( L \) in space, then \( h = (g(f(x))) \) is the subtraction of a trait by which the one is made the same and the local is identified in the global \( R^3 \). If we go the other way by the inverse of \( g^{-1} \), then we begin to put back the object into the diagrams, not as representations in space, but as the fragmentation of a presentation. I will not present this theory or problem here, but am concerned uniquely with drawing out the implications of this schema within the classical theory of links.

We condense the details in the passage from global to local in Freud’s treatment of hysteria in the *Butchers Wife* into three times:

1) the butcher’s wife has a desire to have an unsatisfied desire;
2) she identifies with her husband via a woman she suspects him of having an affair with;
3) she identifies with the woman via the smoked salmon.

These three statements set the framework for what Freud calls ‘hysterical mimesis’ or ‘hysterical identification,’ which we call simply a mode of *Duality* or *Originary mimesis*. It is important to recall that Freud insists that ‘hysterical imitation,’ or identification in general, is not simple imitation, but what he calls ‘assimilation’ (*Gleichstellung*), an identification between diagram and object, mask and actor, that involves an investment of libido that does not allow it to be easily removed.
Each statement above can be viewed as corresponding to a major operator of the analytic clinic:

1) A Representative of Representation (Vorstellungrepräsentanz) of the dream presents the stamp of the unconscious in a denegation—‘I want to have a party that I do not want to have.’

2) A global identification between three people: the wife, the husband, and the young woman, all of whom are locked together in a ‘love triangle’ or ‘triskel.’

3) A local identification with an object a: behind the smoke—or veil of gauze—lies the salmon that the butcher’s wife identifies with in a substitution for her husband’s lover and a displacement of her caviar sandwiches.

It can be proposed that if the first two global hypotheses are Freud’s, the local identification with an object only takes its full weight in the reading of Lacan.

6§ Determining the Identity of the Link in a Structure

Let us determine here a lattice of all the possible manners of two linking components \(2^2 = 4\) in a diagram \(f: L \rightarrow S^1\):

![Diagram of linking components](Image)

**Space of Diagrams with Orientation**

Here we have diagrammed a space of potentials: the structure of any possible two-component 2 crossing link \(L\) in a scale of links, or lattice, where one moves from one link to another by reversing an orientation. Thus, in going from the link \((-1)\) at the top by 0.1 we reverse the orientation of \(a\). In so doing, we pass from the negative linking number \((-1)\) to the positive linking number \((+1)\). Or again, in going from the bottom link \((-1.0)\) to the right via 0.4, the orientation of \(a\) is reversed. It will be remarked that the classical theory of knots and links does not distinguish all four
links since it assumes that those links marked by $\partial$ can be subtracted out as being the same modulo isotopy. Thus, the classical theory of links collapses this structure to $2^2 - 1 = 3$ diagrams with opposed linking numbers. Taking $\{+1,-1\}$ the classical theory of links only recognizes:

$$\{ \}$$

I want to show how, by leaving in the diagrams $\{+1,\partial, -1,\partial\}$ of the excluded subtraction, we introduce a structural theory that determines in a more precise way the identity and existence of the link. Classical link theory begins with the two: it begins with the oppositional pair $\{+1, -1\}$, then runs into predictable difficulties when it tries to show how these two are the same but not one, i.e., they are symmetric objects allowing no movement by isotopy from one to the other. For example, the temptation to call the two $\{+1,-1\}$ links ‘left’ and ‘right’ or ‘mirror images’ has, from Tait and Thomson forward, lead to an ambiguity of terminology from which there is no escape (Tait, 1877). I will not have time to enter into this problem at this point of my presentation, rather I will only concentrate on what are the implications in beginning differently: not with two that are symmetric or that we try to make one, but one that is two. To do so, I will show how the identity of the link can only be introduced by reconsidering diagrams $\{-1, -1,\partial\}$ as a One defined by a *Triple Point Movement*:

24

Here, the *Duality* between $-1$ and $-1^*$ is defined as the change of surface $X$ and void distinctions $A$. Though it is possible to determine different 3-dimensional diagrams of this one projection, we will only use one type of triple point movement in our presentation called a ‘Hybrid Knot Movement’ that we adopt from the working of J. M. Vapperealp.25
Topologically, we will show how this Triple Point Movement introduces a duality between the two link presentations {-1, -1} determining their identity as One but not the Same modulo isotopy. To determine the object of a link theory in terms of an equivalence class, we must add this movement to the Reidemeister moves.\textsuperscript{26}

7§ Triads & Duality

One of the simplest ways to recognize the importance of all four poles of the lattice structure of the link is to notice that despite the linking number of each link, the gyrations of the components are different from station to station:

Lattice of Link Diagrams with Triads
To distinguish each link we determine a triple that determines the gyration ‘g’ of each component. Call this triple the generalized linking number \( \text{lk}(z g(x,y)) \).

To achieve the lattice of linking Triads, we would require sixteen diagrams. We only begin with the first eight on (-1), the other eight on (+1) being symmetric. First, we interpret the gyration pair (x, y) as a surface/void distinction on the link where X and Y are surface indicators and A and B are void indicators. Thus, placing these indications only on the (-1) diagram we have:

![Diagram](image)

Surface Indicators of Diagrams

What this interpretation of the gyration pair (x, y) effectively does is to provide the closed curves with a surface. In an initial probe, such an interpretation may be compared to putting a Seifert surface onto the links: draw a film covering the top \(-1(+1, +1)\), while the bottom \(-1(-1, -1)\) draw the surface to the interior and extending around the link leaving A and B to mark voids.
The achieved twisted lattice of all eight -1 diagrams is:
Commentary and Monstration on the Lattice

1. It is important to note that the vertical movement on the lattice is a discontinuity signaled by the need for the Triple Point Movement that indicates complementation and dualities between a One that is 'not quite the same.'

2. The calculation of linking on a simple set of \{+1, -1\} modulo isotopy does not suffice to identify the link or determine its existence in a precise way. To do so requires distinguishing the obstacle between -1 and -1/Dual and the Complement including the gyration of (+1,+1) or (-1,-1) indicative of the void/surface distinction.

3. Traveling along the lattice to the same color corners, one proceeds by a continuous movement of flyping a component f(a) or f(b). This movement could also occur through a series of Reidemeister moves.

4. The lattice itself is twisted and forms a link diagram, i.e., if one follows carefully the edges of the lattice it is not a cube, but an Escher Cube, which is nothing other than a link.

To conclude, we construct an example of the Triple Point Movement which permits a discontinuous transformation from the top most link \(-1(x_1, y_1)\) to its Dual \(-1^\ast (-x_{-1}y_{-1})\). The sequence of movement is composed of all Reidemeister moves, except step 5. A Triple Point movement on two separate strings:

[Diagram of Triple Point Movement]
I have undertaken in this paper to describe a placement problem that is crucial to both a topological and analytical theory. I have been careful to not accept as valid any of the techniques or presumptions that would normally be accepted as standard in the contemporary mathematical theory, while not de-supposing the merit or rigor of such methods.

I have barely calculated anything at this point, but have tried to show how there is something like a style in the contemporary mathematical theories that consti-
tutes an extra-mathematical set of choices that is not so much a paradigm, but an inhibition to the materiality and locality of the signifier. Call this inhibition, at the minimum, Lebinizian, and at the maximum, a form of sleep that can be approached clinically.

Whatever formal theory one may be trying to construct, whatever the sophistication of techniques, it is necessary to analyze a point of diffraction that occurs the moment one works a theory of the signifier and letter into a mathematical practice. I have called this point of diffraction anti-Leibnizian.

The place and implications of an anti-Leibnizian reading were indicated in §1 Generalized Placement—but I did little more than locate the problem when I situated the contingency of the diagram with regard to a theory of the signifier. Let me simply say that a fuller analysis must be left until a later study.

For the moment, the most I can do is to indicate the direction in which the problems raised by this article can proceed to be resolved. This may be summarized thus:

1) Is the link sensitive to its diagram, i.e., does it depend upon its projection?

If one supposes that isotopy suffices to determine an invariance of form, then the answer is ‘no.’ But if one can show that there are links with the same linking number that cannot be isotoped into each other via the Reidemeister moves, then we must respond ‘yes.’ We showed this with the non-oriented link -1 and its Dual *-1.

2) If the link depends on its projection, then:
   
   a) The identity of the link can no longer be strictly determined by a binary linking number lk(x, y), but requires a triple lk(x (y, z)).

   b) The oriented crossings governed by Maxwell’s right hand rule no longer suffice to determine either the existence or identity of linking; on the contrary, what is required is a crossing number determined by a surface void/distinction:
c) The distinction of mirror symmetric links [+1,-1] can be established only if a prior question of how to regulate the problem of duality [-1,-1] is resolved. Thus, we may in any respect consider that a theory of linking exists prior to putting any orientation on the components or symmetry between them.

3) The Link can be identified only if it can be determined in its structure; in other words, if one can show what makes it different not only from knots, tangles, and locks, but from itself.

a) Call this interior difference of the One, Duality: it is the Representative of the Representation constituting a local Freudian Identification in the same way an actor identifies with a mask he can not take off, or a mime mimes herself miming; or a knot identified with a diagram is a one that is two.

b) The moment -1 is differentiated from its Dual *-1, one is made the same and is two in the production of an object in the discontinuous passage of a triple point.27

c) Call the exterior difference with ‘all the rest’—non-links: locks, tangles, and links—the global differences between types. Whereas the interior difference of the One as the Same is the Dual and results in the bifurcation of the type.

The principal result of this article is not to have invented anything new, but to have restated a problem in a way that is precise enough to develop the consequences.

I landmark this problem here with a conjecture on the clinic I call:

4) The Vappereau Conjecture: The only objects, whether Knots, Locks, or Links, not undone by a Triple Point Move are Links.28

If true, it provides a criterion for the identification of Links in a way that is no longer reliant upon isotopy equivalence. Thus, it would permit the following conjecture to be responded to:

a) The unknotting number requires a projection dependent theory where:

\[ u(k) \leq \min u(k) \]

A theory of links today must resolve the following problem: if the existence and identity of linking, in the sense strict, cannot be established by the oriented crossing number of diagrams, then we must begin with a structural theory of un-oriented linking number of a diagram. Given the first un-oriented link, place a surface on it as follows:

\[ \Sigma(1 + 1/2) = 11 (\text{absolute value}) \]
Now, designate the crossing number exactly as one had done with the oriented diagram. It will be discovered that the un-oriented linking number is sensitive to a change of presentation and cannot be undone by a triple point move.

There are different presentations of one diagram: it can be both $\{+1, -1\}$. Unlike the oriented theory of links that asks how the same two are one, the un-oriented theory asks how one is the same two.

**Appendix I—Leibniz’s Clinical Analysis Situs**

The *manner* by which the Same—considered as an equivalence class of predicates—never suffices to establish an identity of the One we call a *style*, or clinically speaking, a *symptom*. It is important to bring out, however briefly, the correlation between *symptom* and *topology* since this shared perspective did not begin with Lacan and has led to confusion.

Already Leibniz declared that disease can be considered analytically, based upon symptoms, or synthetically, based upon causes ("*una Analytica per symptomata, altera Synthetica per causas*," Leibniz, 217), and asks if “all symptoms are simple illnesses” since analysis is “a general healing method, which is to the pathological synthesis what algebra is to the elements of geometry.”

Leibniz’s conception of truth is based on his principles of identity which are based on his *principle of sufficient reason*: it is only on the assumption that the Same is One that Leibniz sets up the possibility of a regressive analysis that proceeds by assuming the truth of what it wants to prove: that two can be the same, at least formally, and from this infer identity of the one. This is his principle of the *identity of indiscernibles* and the basis of his *analysis situs*.

The project of Leibniz traces both a formal symptom and dilemma inherited by modern mathematics: the converse of the *identity of indiscernibles*—the *indiscernibility of identity*—is only thought pathologically: if one begins with two, two leaves or
two sexes, then there can be no indiscernibles by nature, since no two can ever be alike or truly one materially. For Leibniz, if two can be one it is only on the conditions that they be the same in a formal system of equivalence. Thus, any reference to a real is extra-mathematical: what Leibniz calls a ‘pathological synthesis,’ an error of logic or a geometry without an algebra.

Experiment: Trace a circle on the page. How many circles did you trace One or Two?

There are only two strategies this author knows to take here.

1) The Same is One: you suppose a circle is only truly a circle in a formal theory, then the circle you just traced with a compass on the page is not the same and can only be a second symptom-circle defined as an exception to the form of the first circle. The first absent formal circle is only supposed, even though there may be a desire to prove its existence in a regressive analysis and in the use of the “healing method” Leibniz calls algebra. Call this strategy Leibnizian: it begins with two, then tries to show, through a regressive analysis, how the two is the same in a formal theory by trivializing any material difference.

2) The One is the Same: you pose that one circle is the same only in fact: from the choice of conditions by which it is presented. For instance, in projective geometry one circle is never just a circle, but always dual in consideration of a pole or polar presentation and the equation fixing its signification. This difference of the same does not arise because it was badly drawn on the page, but in the consideration of a choice of presentation and writing. Thus, intrinsically, one circle is different in principle, but is only the same in fact, experimentally, in reference to a choice of presentation. Call this strategy anti-Leibnizian: it begins with One, then shows how it is the Same on the basis of a material presentation.

For the anti-Leibnizian, a mathematical or case presentation is not a secondary material deviation from a true form, any more than a symptom is a deviation from the norm: both are manners of discerning and writing how the one is the same with a difference. When a woman substitutes a hat for her husband, or a diagram for a knot, she is very well substituting on discernibles—counter to Leibniz’s wishes—though she may still seek a normative therapist for a principle of sufficient reason and a regressive analysis of a symptom.

Yet, if we begin with the premise that the one is the same, then we can always substitute one for the other in a manner that does not repose on reducing a symptom to a regressive analysis and the search for good form, but a division of the one itself. For example, we must ask if a truly good text is ever the same. No doubt, if it is One, it can be Same, but this does not imply the Same is One: there is a text that is productive of a division not contained by the Book. Not only will no regressive analysis find the cause of a difference of the One, but its symptom and clinic are not
responded to by therapeutic questions: it is not a question of returning a deviation to a norm of life or a regressive analysis, but of working with a different style of negation and a structural cleavage.

In recognizing how the One is the Same, but cloven, we discover the anti-Leibnizian Freud: counter to a psychology aiming to reduce identity to a unification of what is the Same (the soul), Freud determined identification in a repetition (Wiederholungszwang) of what is One: the Einziger Zug. No doubt, there was a period where the psychoanalytic symptom was treated in a regressive analysis of the cathartic cure, but later not only would Freud discover its principle of sufficient reason—the Phallus—but show how its lack founds the basis of the clinic. Said otherwise, the Phallus as Form of the Same veils the problem of Structure, more specifically, the structure of castration. Beyond the hypnosis of Form and a regressive therapeutics, analysis reposes on a reading and writing of this unary trait in a theory of the signifier. In the Sinthome, Lacan proposed Freud’s Einziger Zug could be formulated topologically as a triskel:

![Triskel](image)

It is a question of how three is in two, or more precisely, how this third reduces two to one. For Freud what makes a man a man is a phallus, but what makes a woman a woman is a phallus also, which is the scandal of reducing two to one in the same.

My short introduction asked what makes a knot a knot? What is this repetition of one in the same that is bypassed in the classical theory of knots? The problem of Symmetry is aligned, but not a problem of Duality. Just as how the problem of Two become One in the Same is not how One becomes Two in the Same. The former constitutes the phallic stage, while the second is the introduction of the Oedipus by which Freud discovered castration in the Einziger Zug.

Without denying the narratives of Mama and Papa or the cultural situation in which analysis today is asked to give regressive reasons for everything—from pipi, caca, tics, and political runny noses—there is a practice of an experimental topology that can only constitute a progress for analysis.

Appendix II


Robert Groome, “Generalized Placement PartII: Reading the Knot” (circulated pre-print 2010).
Acknowledgements

These series of articles are dedicated to the late G. Chatelet and J. M. Vappereau. Without their work and critique this would not have been possible.

Notes

1. See 2.1§ below.
2. Einstein acknowledged that his relativistic approach mirrored Winteler’s linguistic courses that defined a structure in terms of invariants and variants. Although it is well known that Jakobson equates the ‘differential elements’ of language discovered by Saussure with the ‘elementary quanta’ of phonemes, his reference to Winteler’s work is often bypassed. See Jakobson, 1972.
5. Koyré writes, “Indeed, an experiment—as Galileo so beautifully expressed it—being a question put before nature, it is perfectly clear that the activity which results in the asking of this question is a function of the elaboration of the language in which it is formulated. Experimentation is a teleological process of which the goal is determined by theory.” Alexandere Koyré, "An Experiment in Measurement," Proceedings of the American Philosophical Society 97 (1953): 222-237.
7. “Car le savoir accumulé dans on expérience concerne l’imagination, où elle vient buter sans cesse, au point d’en être venue à régler son allure sur son exploration systématique chez le sujet. [...]. L’expérience en ceci ne donne de privilège ni à la tendance dite ‘biologique’ de la théorie, qui n’a bien entendu de biologique que la terminologie, ni à la tendance sociologique qu’on appelle parfois “culturaliste.” [...]. A vrai dire, si l’analyse confine d’assez près aux domaines ainsi évoqués de la science pour que certains de ses concepts y aient été utilisés, ceux-ci ne trouvent pas leur fondement dans l’expérience de ces domaines, [...]” Jacques Lacan, Variantes De La Cure Type (Paris: Seuil, 1966). It is important to remark that the word used by Lacan in the original French, ‘expérience,’ is underlined here and translated by myself as both ‘experiment’ and ‘experience’ in English since it is not by simple experience, but by experiment that the growth and struggle of science transpires. The subject of science, and by implication psychoanalysis, is not experiential, but experimental. In misrecognizing the experimental dimension of the subject, the scholar Bruce Fink’s unilateral translation of the French word ‘expérience’ into the English ‘experience’ trivializes the text and makes it impossible to recognize the clinic as anything other than experiential

8. See page 61 for diagram of homotopy.


10. To the objection that the existence of the knot can be determined by its complement in space, it must be recognized that this type of existence—as being different from a closed curve or trivial knot—is not the same thing as to calculate the unknotting number with regard to its presentation. Moreover, to identify a knot by calculating its complement only works for proper knots, i.e., knotted single component closed curves. Links and Locks, for example, do not have calculable complements sufficient to identify them much less determine their existence.

11. For a presentation that relies only on a logic of the traits of the diagram to establish a crossing, see R. Groome "Generalized Crossing Numbers: A Theory of Triple Points," (2002).

12. I have no intention here of giving a development on Leibniz’s Principium Identitatis Indiscernibilium. Rather I would like to use it to open up a set of logical problems that arise once the unidirectional formula of invariance is generalized to include its converse—the indiscernibility of identity. Stated in predicate logic, Leibniz’s principle of the identity of indiscernibles may be written ∀F(Fx ↔ Fy) → x=y where the statement may be parsed in ordinary language as: if, for every property F, object x has F if and only if object y has F, then x is ‘equal’ to y. The assimilation of an equality to identity may be assumed, but requires one to entertain the applicability of properties P to objects x, or to consider the truth and falsity of values P(x) of a predicate, before the objects of the domain are clearly distinguished. This move is not taken in a formal theory of knots. We will call this not yet distinguished or un-named object a Thing of knot theory in Part II of our presentation.

13. Again, we may drop the quantification on the predicate D only if we assume the notion of equality underlies the space (domain) and the predicates we are working in.

14. The choice of a particular link or a class of movements may be defined as a Representative of the Representation (*Vorstellungrepräsentanz*), i.e., a model for a particular type of knot. For example, the choice of a particular cloverleaf may be used to determine the type for all clover leafs. It is important to determine whether such a choice and particularization of a type can be trivialized or not. See §5: The Love Triangle of the Butcher’s Wife.

15. It should be remarked immediately, if the arrow-orientations are left off the links, then they are of the same isotopy-type.

16. P.G. Tait (1877) first introduced this term in borrowing from the Scottish dialectic meaning to ‘turn inside out.’

17. See Appendix II.

18. See section §6.
19. Counter to Leibniz, and known at least since Parmenides, the Same is One—Leibniz’s identity of indiscernibles—is not equivalent to the One is the Same—Leibniz’s indiscernibility of identity. See Appendix I.

20. Two terms which contain each other and are nevertheless equal may be called ‘coincident’ in the manner of Leibniz. For example, everything that is denoted by the term ‘triangle’ may be denoted by the term ‘trilateral,’ yet the two have a different sense and may be used in different manners. Yet, it is well known that the whole substitution principle salva veritate falls under the critique of Frege’s Über Sinn and Bedeutung (1952) for failing to make the distinction between ‘mention’ and ‘usage.’ Here we have the linguistic version of complexity involved in not making the distinction between sign and object or diagram and knot.

21. These questions have a long, perplexing tradition and are far from being resolved. For example, Leibniz himself shows that the substitution principle breaks down salva veritate on reflexive conditions in which the manner of designating what is one is brought into play. For example, Leibniz states: If A is B and B is A, the A and B are called the same. Or, A and B are the same if they can be substituted for one another everywhere (excepting, however, those cases in which not the thing itself but the manner of conceiving the thing, which may be different, is under consideration; thus, Peter and the apostle who denied Christ are the same, and the one term maybe substituted for the other, unless we are considering the manner in the way some people call “reflexive”: for example, if I say, “Peter in so far as he was the apostle who denied Christ sinned,” I cannot substitute “Peter” and say “Peter, in so far as he was Peter, sinned.” E. Bodemann (1966).

22. Similarly, a fundamental clinical problem is posed the moment a medical doctor examines a patient and proposes there is nothing biologically wrong with the patient—no lesion in an organ or chemical imbalance—even though there seems to be a disturbance in the functioning of organ: an arm will not move or an eye is ‘blind.’ If p implies q, i.e., if I have a lesion in my brain, it implies that a motor activity will be disturbed; but the converse, ‘q implies p,’ if I have motor disturbance does not mean there is a lesion in my brain. To think so, would be to confuse implication with equivalence.


24. The mathematical literature shows a similar use in the Yang-Baxter constructions: a term.

25. In ”Noeud” (1997), J. M. Vappereau introduces a second movement necessary to the general theory of links which he calls Gordians. This generalization need not be considered in my presentation here.

26. In so doing, we begin to shift the global-local correspondence of the object and diagram from 3-d space to 2-dimensional plane to a projective space and projective plane. At which point, the reference to the Borromean is put into play since it determines the connectivity of the projective plane in the same manner that closed curves determine the connectivity of the sphere.

27. Recalling that Lacan proposed the butcher’s wife identified with the salmon behind the veil.
28. On the communication of this statement to J.M. Vappereau, he informs me that he considers it a theorem with regard to p.278 in his Nœuds. With regard to 4(b) below, I have not been able to confirm this result with regard to non-alternating knots and presentations.

29. Duplex Methodus tractandi Morbos, una Analytica per symptomata, altera Synthetica percausas. Considerandum est, omnia symptomata esse morbos quosdam simplices, semper enim sunt laesae functiones, sed quia una functio laesa facit plures [alias] collaei, hinc saepe causa pluriurum symptomatum unica haberi potest. Si laesio functionis non sit perceptibilis per se non appellatur symptoma. Methodus tractandi per symptomata, infinita esset si omnes eorum combinationes enumerare vellemus. Sunt quaedam signa bonae malaque constitutionis, quae symptomata dici non possunt, ut color, urina, juvantia et laedentia; vera tradenda est analysis, seu ars tium in signa inquirendi, tum ex signis concludeendi morbum. Tradenda est synthesis post subjicendum specimen analyses; seu Methodi medendi generalis, quae habet se ad synthesis pathologicam ut algebra ad Elementa Geometriae. G. Leibniz, De scribendis novis medicinae elementis, 1680-1682

30. The analytic theory of Homosexuality does not repose on a definition of Same-Sex relations, but how the Same—or Homo—is radically Hetero. A woman who chooses a man as a sexual partner is just as homosexual as a man who chooses a man as a sexual partner; inversely, a woman who chooses a woman as a sexual partner is just as heterosexual as a man who chooses a woman. In either case, the same never suffices to stabilize the conditions for what is one.

Bibliography


Groome, R. T. "From A Set Theory of Numbers to a Topological Theory of Numerals." Sclinic. PLACE, n.d.
—. "Generalized Crossing Numbers: A Theory of Triple Points" (circulated pre-print 2002).
—. "Generalized Placement Part II: Reading the Knot" (circulated pre-print 2010).
—. Problèmes cruciaux pour la psychanalyse. pre-print, 1966.
Leibniz, G. De scribendis novis medicinae elementis. 1680-1682.